NUMBER SYSTEMS AND COUNTING

INCA QUIPUS: AN INVENTORY SYSTEM OR WRITING?

The Inca lived in South America about five hundred years ago. They were the rulers of a large empire, which had its center in Peru, and extended from Chile to Ecuador on the Pacific coast between the ocean and the high mountains of Andes. The country was long but narrow, and the distances between cities were long. So the emperors built good roads paved with stones to connect them with each other. Yet the traffic consisted of people and llamas walking only because there were no horses or oxen to carry loads in America. The only beast of burden was the llama which is a rather small animal and rather slow also.

Messages from one city to another were carried by messengers who run from one post to the next like in relay race. The people in the Inca empire did not 'write' messages, however, but the information was sent in form of knotted cords which were called 'quipus'. Another method of coding messages was also known, namely painting beans. This is known from old vessels which were decorated with pictures of running men carrying small bags containing marked beans.

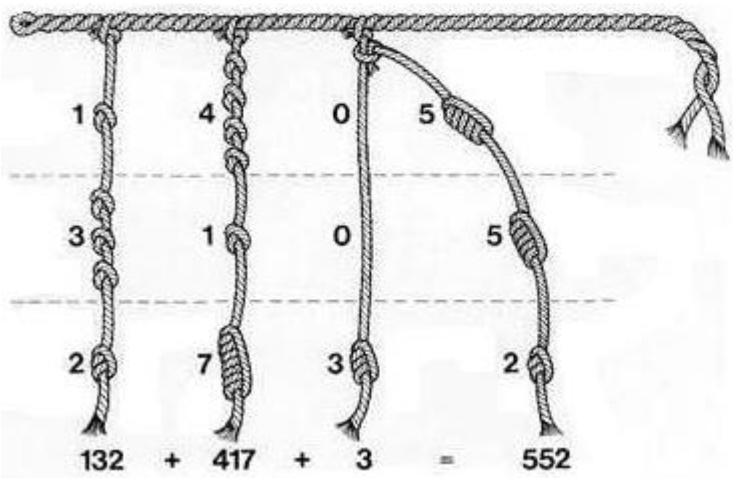
The Inca emperors were not unlike modern rulers as they also collected lots of taxes from the people. There was no money in the empire but taxes were collected in quantities of maize and potatoes, textiles, ceramic pots, llama wool, shells, and precious stones and metals. All the gold was sent to the Inca emperor because he was believed to be the son of the sun, and as gold has the color of sun, it belonged to the Inca. There were large storages where the tax goods were kept and distributed according to orders of the Inca. The storage keepers recorded everything with quipu knots and sent them to the capital. The quipus were collected to a central inventory where trained quipu masters could read the information. For the purpose of taxation and administration, census information was also kept: information about births and number of people in different regions.

After the arrival of Spanish conquerors the knowledge how to read quipus was lost. As it was a kind of secret knowledge, mastered only by some high officials, the skill died out together with them. This is also the reason why quipus are a riddle for contemporary scientists. Yet, a humble version of quipus has survived: on the island of Puna in Ecuador, people still keep records using knotted cords.

A quipu is an assemblage of colored knotted cords: there is one thick main cord, and lots of small cords are attached to it. Some very big quipus with even 2,000 cords have been found. The cords are of different color to mark different objects. The quipu contained numbers and labels, the number of knots indicating the amount of items in the storage. The counting was based on ten, like in our decimal system. Everything in the quipu had a meaning: the coloring, the way the cords were connected, the spaces between the cords and the knots, the placement and number of the knots and spaces between them. But those exact meanings are not known anymore.

Most scientists believe that quipus contained only numbers, and in order to understand a quipu message one had to remember several rules. Thus, they were a sort of advanced memory aid. However, some scientist argue that as the quipus were used as memory aids in reciting stories, genealogies and prayers, they could have been used for expressing sounds of language, that means writing, as well.

It is clear, however, that the writing and symbol systems in the Andes region were quite different from ours. The textiles from that area have complicated geometric designs which all have specific meanings. From the patterns of clothing and decorations of a person in the Inca empire one could immediately see her home town and status.



PRACTICE

Compare the quipus with the Ogham writing (described below). There is some similarity in the idea, and you could try to develop it. Try to design a writing system using knots. Hint: replace the Ogham notion of 'above' with one knot, 'below' with two knots and a long line with three knots, leave a small space and make as many knots as there are lines.

Or an alphabetic writing can be construed by giving each letter a number, so that A=1, B=2 and so on up to Z=26. Then the word INCA would be on knots: 9 14 12 1

Explore the possibilities further: To make "writing" with knots without the need of spaces, a combination of three groups of one, two or three knots covers 27 characters. How would it look like? On the other hand, a syllabic system with about 50-60 different characters (numbers) is also possible.

THE MAGIC OF KNOTS

For what do people tie a knot in the corner of a handkerchief? To remind themselves of something important, isn't it? They "bind" that thing to be remembered into the knot so that it could not run away from the memory. This magical meaning of knots stems from the ancient Babylon where witches made charms of knotted cords already earlier than three millennia ago. The Ancient Greeks and Romans perhaps got from Babylonians the idea that knots have magic influence because of their power to bind and strangle. The Greeks made for example love magic with knots, and wrote binding spells to bind the desired person to oneself for ever.

A typical binding spell against enemies sounded like "I bind So-and-so to such-and-such action." To bind a beloved:

"I bind you, Theodotis daughter of Eus, by tail of the snake and by the mouth of the crocodile and by the whiskers of the cat, that you may not be able to make love ever with another man except me alone Ammonion Hermitaris."

The tradition of knot magic is still alive today on the Greek islands. Once a young man called Manoli from a rich family fell in love with a girl from a poor family. Manoli's family felt that as he had eaten at the girl's house, someone there must have put love potion into his food. Both families were against marriage but the young couple eloped. After a few days they returned and were married. But Manoli got sick and was bedridden for four months. The priest of the village came every day to bless him but it did not help. Finally Manoli's family went to ask for the help of a witch. The witch explained that someone who had been at the wedding ceremony had uttered magic words and had tied three knots in a string, thereby acquiring power over the couple. This magic spell could only be broken by a curing ritual and repeating the wedding ceremony. The couple was remarried, and Manoli recovered. But the marriage was not happy, and a few years later they divorced.

The most famous knot in history was kept in the town of Gordion. It was an enormous knotted rope with a magic spell on it. A young warrior and king of Macedonia, Alexander, was shown the knot and told that the man who could untangle it would conquer Asia. Alexander saw that trying to untangle it by hands would be a futile effort. As he was young and brave, he took his sword and swiftly cut the knot into halves. He then went to conquer huge areas from Greece to India, by which he earned the epithet 'The Great' from later generations. But his fortunes show that rough methods have similar consequences: Alexander died young, and after his death the hastily made empire was dissolved into parts.

KNOTS AND MEMORIZING

The East-African Makonde tribe measures age of children with knots. Each month at the full moon, a knot is tied to a rope. When the baby is twelve months old, another rope for years is started. In the same way, days were counted. When a man leaves his home to make an eleven day trip, he gives a rope with eleven knots to his wife. He shows the first knot and says: "This one is the day when I leave. Tomorrow I will travel, and the next two days also." He shows the fifth knot and continues: "This day I'll arrive there. I will stay one day, and on the seventh day I'll depart. Don't forget to open one knot each day, because when the tenth day comes you have to start to prepare a meal for me. I want it to be ready the day I come home."

By the Yoruba of Nigeria, cowrie shells are tied to a string to convey a message. One cowrie shell means 'defiance and failure', two placed together mean 'relationship and meeting', and three placed apart 'separation and hostility'. Six cowrie shells in a string makes a love letter, for in the Yoruba language the word 'efa' means both 'six' and 'attracted'. A string of eight shells returned to the sender stands for 'I feel the same' since 'ejo' means 'eight' and 'agreed'. Think about your next Valentine!

MESSAGE STICKS

Message sticks function much the same way as knots on a cord. People used to mark down days by notching a hole on a stick. Small farmers in Europe made notches on sticks to record their debts to landowners, or the number of days they had worked for pay. This is called a tally stick. Among the Maoris of New Zealand each incision on the stick recalls the name of an ancestor. Their message stick was same sort of a memory aid as the tied handkerchief.

In olden times, the distinction between writing and numbers was not that clear. Same kind of signs or markings could be used for letters, words or numbers. Some modern words remind of this: a bank teller counts money but a storyteller works with words. A bank account has numbers but an account of events contains words.

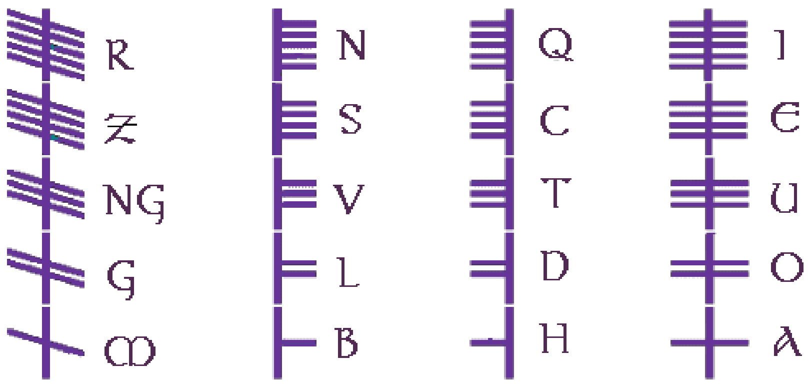
OGHAM WRITING: LETTERS LOOK LIKE NUMBERS

Here is a picture of a stone which was found in Ireland. Does it look like a tally or a message stick? Of course it does. In fact, it was thought to be graffiti until last century when people started paying interest in odd-looking carvings on stones. They were finally recognized as writing when an old manuscript with different styles of Ogham was found. It provided the key to the alphabet which can now be read.

Ogham writing was used by Celtic peoples in Ireland and Scotland more than a thousand years ago to write languages called Irish and Pictish. When it was replaced by the Roman alphabet in the 7th or 8th century, it got completely forgotten. Ogham inscriptions are not the easiest to read, as they are usually very short, and the direction of writing is sometimes from right to left or from bottom to top. Another problem is caused by the vowels which may have simply been left out.

It is believed that the inventor of Ogham alphabet knew about the Roman or Greek alphabet. But the signs do not resemble any other alphabet, and the letters are arranged in an ingenious manner. It looks like a systematic effort to devise an easy alphabet: just look at the vowel group! There is also speculation that the Druids who were the priests and magicians of Celts, used finger signs to instruct pupils in secret rites, and that the Ogham signs were developed from those finger signs.

Notice, that in the Ogham alphabet some English letters are missing. If you play with the Ogham script, these letters have to be replaced by other letters. For example, the following substitutions can be made: F=V, J=I, K=Q, P=B, W=U, X=CS, Y='UI'.



PRIMITIVE COUNTING AND BODY COUNTING

In many societies around the world only three number words are used: 'one' or single, 'two' or a pair, and 'many'. This kind of counting is used by tribal peoples who live in small groups in rainforests or desert areas. They produce their food by gathering plants, nuts and shells, hunting and fishing, and perhaps keeping small gardens, They do not collect much property, nor do they trade many items.

The Bacairi live in the Mato Grosso rainforest in Amazonia in Brazil. They have number words only up to two. Their word for 'one', tokale, comes from the word 'bow'. A man has one bow but many arrows, thus the word for two, ahage, means many. With these two number words, the Bacairi count both their hands and feet:

A person starts counting with the little finger of the left hand and says 'tokale'. The he grasps the ring-finger and joins it with the little finger and says ahage. He goes over to the middle finger and says, holding it separately beside the two joined fingers, 'ahage tokale'. Next, index finger is joined with the middle finger, 'ahage ahage', that is two pairs. 5 is 'ahage ahage tokale', the thumb of the left hand. Little finger of the right hand is added to make six, 'ahage ahage ahage'. After this the Bacairi continues with the fingers saying mera (=this one) for each. In a like manner he touches the toes of the left and right foot and each time says 'mera'. (This is easy as long as he does not wear shoes.) When a Bacairi comes to 21, he grasps his hair and pulls it apart in all directions!

As you can see, they basically count in pairs, and 20 is the maximum they care about. Try Bacairi counting and make a picture of the hands and mark the numbers!

The Yanoama people from another part of the Amazon rain forest count up to 3:

1 mahon, 2 porokabo, 3 prukatabo

All amounts starting from four are called 'pruka' which means 'much', or pruka-pruka (=very much). Even without number words, the Yanoama can make a difference between smaller and larger amounts. If one of them has 20 arrows standing together and someone comes to take one during the owner's absence, he will notice the change at once upon his return. How do you think this is possible?

Discussion: Why do these people have so few numbers? For what do they need counting? Do you think they could learn more counting if they went to school?

PRACTICE:

Write the result in Bacairi:

2+2 =	2+4 =
1+0 =	1+2 =
3+3 =	3+5 =
3-1 =	14+9 =
8-2 =	

PRACTICE 2:

The Coahuiltecan Indians from Texas have only six number words, namely 1, 2, 4, 5, 6, and 20. They compile other numbers by adding and multiplying these six words. Look how larger numbers are got as results of small numbers:

2+1 =	
(2+1)*2 =	(5*2)+1 =
4+(2+1) =	4*(2+1) =
4*2 =	4*(2+1)+1 =
4+5 =	4*(2+1)+2 =
5*2 =	5*(2+1) =
5*(2+1)+1 =	
5*(2+1)+2 =	20+(5*2) =
6*(2+1) =	20*2 =
6*(2+1)+1 =	(20*2)+(5*2) =

Can you see any rules in forming the numbers?

How many different number words do we have in English?

BODY COUNTING: CHINESE NUMBERS

The English word 'digit' still reminds of an earlier time when finger-counting was a rule: it means both a finger or toe, and an Arabic numeral. Also the Chinese number signs are assumed to have developed from counting with fingers. This is how it looks:

一 二 三 四 五六七 八 九 十

BODY COUNTING: THE PAIELA FROM PAPUA NEW GUINEA

The Paiela are a small tribe which lives in the highland Papua New Guinea. New Guinea is a large tropical island where hundreds of small tribes live separated from each other by jungles and mountains. The Paiela grow food in their gardens, they hunt forest animals, and they keep pigs. Pigs are of high value: they are eaten in feasts and given as presents. The Paiela do not use much paper money yet.

The most important event when the Paiela need to count is when pigs are given as wedding presents. The groom's relatives gather 28 pigs for the family of the bride. For purposes of presentation and distribution, these 28 pigs are divided into two equal sets. Fourteen pigs are tied to one long line of stakes for the family of the bride's father, and another fourteen pigs are tied to another line for the family of the bride's mother.

The Paiela count their fingers and the upper body. The count begins on the small finger of either hand. The count then 'ascends' to the thumb, the wrist, the shoulder, and the head, stopping at the nose, where the unit 'one at-the-nose' is summed. A second unit of fourteen is traced on the other side of the body. This time the count 'descends' from the head to the shoulders to the hands, finishing on the small finger of the opposite hand for the 27th count. The hands are then clenched and brought together, the two thumbs knocking against each other, to signal the completion of a second fourteen-count unit, 'two at-the-nose'. The word 'pondo' for 28 is said as the fists join and bump.

The words from 1 to 5 are just number words without any other meaning, but numbers from 6 to 14 are names of actual body parts. The Paiela make a difference between upper and lower parts of the body. The upper body: the head and the arms, is the more important, and it is the part of body to be counted. The lower portion, the 'legs', is considered somewhat dirty, and the lower members of the body are 'not to be counted'.

PRACTICE:

Draw a picture of a person and indicate the Paiela numbers. Find pictures of Papua New Guineans and make your image to look like them. Try Paiela counting yourself. They may be the only people in the world to use a 28-number system!

Paiela numbers and their corresponding body parts:

1	mindi	(small finger)	15	ambi leetsia	other eye
2	lapo	(ring finger)	16	ambi aletsia	other ear
3	tepo	(middle finger)	17	ambi matsia	other neck
4	tukumindi	(index finger)	18	napi payatsia	other shoulder
5	yau	(thumb)	19	napi pilitsia	below other shoulder
6	wataka	wrist	20	napi kitupatsia	above other elbow
7	yanatsia	below elbow	21	napi yanatsia	below other elbow
8	kitupatsia	above elbow	22	napi wataka	other wrist
9	pilitsia	below shoulder	23	napi yau	other (thumb)
10	payatsia	shoulder	24	napi tukumindi	other (index finger)
11	matsia	neck	25	napi tepone	other (middle finger)
12	aletsia	ear	26	napi lapone	other (ring finger)
13	leetsia	eye	27	napi mindi	other little finger
14	ingatsia	nose	28	pondo	two clenched hands knocked together

VIGESIMAL SYSTEM AND OTHER BASE NUMBERS

Today, the decimal system is almost universal. It used to have more competition, even in Europe. Systems based on 20, or vigesimal systems, were strong contenders. As a ten-based system counts only fingers, a 20-based system counts the "whole person": hands and feet. Thus, the word for twenty is often the same as the word 'person'. We already learned one twenty-based system, the Maya numbers.

In Europe, many items have been counted in groups of twenty. In English the word for it is 'score'. In the old Bible translation, the age of a person is expressed in twenty years' age groups (Psalm 90:10):

"The days of our years are threescore years and ten; and if by reason of strength they be fourscore years, yet is their strength labour and sorrow."

PRACTICE:

Calculate these French numbers:

soixante-huit = 60+8 =

soixante-onze = 60+11 =

soixante-dix-huit = 60+10+8 =

quatre-vingt = 4*20 =

quatre-vingt-neuf = 4*20+9 =

quatre-vingt-dix = 4*20+10 =

quatre-vingt-dix-neuf = 4*20+10+9 =

TWELVE-BASED COUNTING

From the ancient Mesopotamia the use of 6, 12, and 60 as important numbers has spread over wide areas of civilization. Even though our system is decimal and based on tens, we have inherited some measurings from the Sumerian use of sixty. We have 60 minutes in an hour, 60 seconds in a minute, and 360 degrees in a circle.

Early Romans had a duodecimal (literally, 2 plus 10, 'dozen') system. It was often used in measurements, and you still measure twelve inches in one foot. We may also buy eggs by a dozen. Twelve was number for grouping things in Scandinavia and Germany about a thousand years ago. Catch of fish was expressed in 'great hundreds', that is, in groups of 120. The Christian 100 was called short hundred or ten tens.

Our twelve hours in a day, and twelve hours in a night is believed to be of Egyptian origin. Twelve was a holy number too: in the Bible there are twelve tribes of Israel and twelve disciples of Christ. Twelve is also an important and holy number in the Buddhist tradition of Asia. The legends mention 60,000 (=12*5,000) teachers of false doctrine, 84,000 serpents, six schools of thought and so on. The great Buddhist king Asoka is told to have given 96,000 kotis of money in 84,000 towns. (The Buddhists loved huge numbers! How are 84 and 96 related to twelve?)

The Maldive Islands lie in the Indian Ocean, and they are so small that they hardly can be seen on a map. The Maldivians used to have a 12-based system of counting until this century. It was quite unique, having (8*12 =) 96 as its terminal number. When sea traders came to Maldives they did not learn properly Maldivian numbers. This may have been one reason for the mistake in the number of islands in the Maldives archipelago: it was believed to be 12,000 instead of the real 1,200. Twelve was the standard number for many things, even a girl's ear was pierced at twelve points along the side. The traditional money from Maldives, cowrie shells, were kept in bundles of 12,000 and 100,000. The cowrie shells were traded to far away countries, even to Africa, where they were very valuable.

PRACTICE: Maldives finger counting

Quick counting was done with the help of fingers: It was needed when fish and coconuts had to be divided up. The method was to count on the fingers two at a time: 2, 4, 6, 8, 10, 12. The fingers are closed when each is said, beginning with the little finger of the right hand; for 10 the thumb is closed and for 12 it is opened again. Tally of twelves is kept in the left hand, a finger being closed for each unit of twelve, beginning with the little finger, then opened again beginning with the thumb if there are more than five units of twelve. Try it!

DUMB AND CLEVER COMPUTERS

We have received our numbers from India. But the very oldest Indian computation system was not a decimal system but a binary one, counting by twos. It was in use some four thousand years ago. It is known from the ratios of weights found in archaeological sites of the Indus civilization: ¼, ½, 1, 2, 4, 8, 16, 32, 64. Each weight was a double of the smaller one. A ten-based system later replaced this binary system, which has however persisted in some measures: the monetary unit rupee was divided into sixteen annas until the 1960s, and regional systems of weights and measures are still based on this.

Some English units follow the same idea: there are 16 ounces in a pound, and food is sometimes sold in half a pound or quarter-pound. The much used American coin is a quarter (of a dollar). One gallon is divided into four quarts, and they again into two pints.

Computers also use the binary system. It is the simplest way of counting of all: just count up to 1, starting with 0 and then 1. Still, the computers do calculations much faster than any human being. They are extremely simple but incredibly fast! The numbers for a computer look like this:

1	10	11	100	101	110	111	1000	1001	1010	1011	1100	1101
1	2	3	4	5	6	7	8	9	10	11	12	13

PRACTICE

Calculations in binary system: do some addition with binary numbers. Examples:

101	1010	10
1	1000	11
_____	____	__
110	10010	101

Check the results from the list above. What about these:

1+11= 10+100= 110+111= 1011+11=

PROBLEM 2:

Size of computer memory is expressed in units of 256 kB, 1064 KB, 4 MB, 8 MB, and so on. How is this related to the binary system?

ARABIC AND ROMAN NUMERALS

Our numbers are called Arabic. Europeans got them from India through Arabs, so in fact, they are Indian numbers. They have been used in Europe only roughly 500 years. Before that time, letters were used as numerals. The Romans made up their numbers using letters and some other signs. The Roman numerals as they are used today have changed from the time of Antiquity. For example, the character D for 500 was originally I), half of a circle for a thousand (I).

Roman numerals in the modern form:

I=1 V=5 X=10 L=50 C=100 D=500 M=1000

Numbers:

2	II	11	XI	20	XX	101	CI
3	III	12	XII	21	XXI	135	CXXXV
4	IV	13	XII	29	XXIX	222	CCXXII
6	VI	14	XIV	30	XXX	412	CDXII
7	VII	15	XV	40	XL	501	LI
8	VIII	16	XVI	60	LX	900	CM
9	IX	17	XVII	70	LXX	1993	MCMXCIII

If you look closely at the Roman numbers you can see that something is missing. Where is the zero? If you try do math with these numbers, it is quite cumbersome. Very often calculations were not done with the numbers but with a kind of abacus, or with tallies, or using counting boards and pebbles. People had to do much more math in their heads than we do. Abacus is still used in Russia, China and Japan, and there are people who are faster using it in addition, subtraction, multiplication and division than you would be with an calculator! This is partly because they do part of the task in their heads. Because they calculate so well they are also able to see immediately if the result is sensible, in other words, if it looks correct.

There are two main differences between Roman numerals and modern Arabic numerals: the zero, and the idea of place-value notation. Place notation is the way to express tens and hundreds by the position of numbers: ones are in the column to the right, tens are left to ones, next are hundreds, and so on. Each power of ten takes one position.

Example: number 35278 has 8 ones, 7 tens, 2 hundreds, 5 thousands and 3 times 105. Number 9967 could be written as $9*10^4 + 9*10^3 + 6*10^2 + 7*10^1$.

These revolutionary ideas of place notation and zero were adapted from India. The Hindus used first only a blank space for zero which was later replaced by a dot, and then a small circle. It was transferred to the west by the Arabs who used a circle with an stroke on top of it. The Europeans were not the only peoples who were so dumb that they did not invent this system. The old Egyptian and Greek civilizations were also able to do without it. Only four old cultures developed a place-value notation: the ancient Mesopotamia (where sixty was the base number, and zero was expressed by blank), India (ten as base number), the Maya (who had a twenty-based system and a proper zero), and China (ten-based, using blank space for zero).

The Roman numerals went only up to a hundred thousand in Antiquity. Larger quantities were expressed by combining signs for smaller numbers: for example a million would be written with ten symbols of hundred thousand in a row.

In a place-value notation there is no limit for the size of numbers, because the same symbols can be repeatedly used. In 44,444,444 the symbol '4' is used for forty millions as well as for four ones, tens, hundreds, and so on.

For practical purposes, like trade and administration, the Roman numerals must have been sufficient. Why did the other cultures develop much larger numbers in that case? The Maya, for instance, used enormous numbers in their calendar and astronomical calculations. In Mesopotamia and China, too, astronomy was of great importance. The science of the universe and mathematics were closely related already at that time.

PRACTICE:

The Soto people live in Southern Africa. They have devised a place-value notation combined to finger-counting. First person indicates ones with her fingers, second tens, and third hundreds. Try this in a group: make numbers 344, 126, 975. Note, that finger-counting can be done in a variety of orders: starting with a closed fist or with an open hand, starting from the left or right hand, thumb or little finger. Standardize your system!

CONCRETE COUNTING

Numbers as we use them can refer to objects, or they can be used in abstract calculations. When you multiply six by eight, the result is 48, but it does not have to refer to any real objects. But when you talk about a duet, a trio, a quartet, a quintet and an orchestra, you refer to a certain kind of an ensemble. Many languages have different number words for different kinds of objects. In English it happens only in rare cases: there are twin children or triplets, single persons or married couples, and a brace of dogs or pheasants. For example Chinese, Thai, Turkish languages and Japanese use these so called classifiers together with the number words.

In Japanese counting is done by object classes: -hon for bottles and -nin for people. The number words get an ending which tells the kind of objects counted. Japanese numbers are:

1 ichi	6 roku	11 juu-ichi
2 ni	7 shichi or nana	12 juu-ni
3 san	8 hachi	20 ni-juu
4 shi or yon	9 kyu or ku	30 san-juu
5 go	10 juu	100 hyaku

When you combine number two and the ending for a bottle, you are able to order two sodas: 'ni-hon'. Sometimes there are small sound changes to make the word easier to pronounce, like for one soda: 'ichi+hon' becomes 'ippon'. The count of bottles continues: sanbon, shihon, gohon, roppon, nanahon, happon, kyuhon and juppon. You can count all long, narrow objects like pencils, trees, or belts similarly.

Other endings are 'mai' for papers, flags, stamps, carpets or magazines; 'dai' for cars, radios, machines or televisions; 'satsu' for books and notebooks; 'hiki', 'biki' or 'piki' for small animals like fish, cats and dogs. Large animals like horses, cows, lions and whales are counted as 'too' (pronounce the 'oo' like in the word 'floor'). For birds you use 'wa', 'pa' or 'ba'.

Example: if you count a litter of kittens, it goes like this: ippiki, nihiki, sanbiki, yonhiki, gohiki, roppiki, nanahiki, happiki, kyuhiki and juupiki.